Effects of Exchange Symmetry on Full Counting Statistics

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We study the full counting statistics for the transmission of two identical particles with positive or negative symmetry under exchange for the situation where the scattering depends on energy. We find that, besides the expected sensitivity of the noise and higher cumulants, the exchange symmetry has a huge effect on the average transmitted charge; for equal-spin exchange-correlated electrons, the average transmitted charge can be orders of magnitude larger than the corresponding value for independent electrons. A similar, although smaller, effect is found in a four-lead geometry even for energy-independent scattering.

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I. INTRODUCTION

Full counting statistics provides the ultimate information on the statistics of charge transport in mesoscopic systems. The first study of this type[1] already provided a surprising result, a (quantum[2,3]) binomial distribution of charges, telling that the flux of incoming electrons is regular (a consequence of exchange correlations) and the scattering events are independent. This result applies to the situation where the scatterer is characterized by an energy-independent transmission and for a constant applied voltage. The modification due to adiabatic pumping[4] leaves the binomial result largely unchanged. Recently, much interest has concentrated on single particle sources feeding devices with individual electrons;[5,6,7,8,9] such single particle excitations are generated by (unit-flux) voltage pulses. Again, it turns out that the full counting analysis remains binomial when the system is driven by such voltage pulses.[7] These findings make explicit that exchange correlations are absent in the (energy-independent) scattering process. In this letter, we demonstrate that exchange effects manifest themselves when single particle sources inject electrons into devices with energy dependent scattering.

The analysis of a setup with energy-dependent scattering and driven with a *constant* voltage has unveiled sensitivity to exchange in the noise but not in the average current.[10] Furthermore, exchange effects in the noise were found in the statistics of two particles escaping from a dot at short times[11] and for two electrons traversing a reflectionless symmetric beam splitter[12]. Here, we show that applying voltage *pulses* in the incoming lead and injecting individual particles produces non-linear transport due to exchange effects; i.e., a twice larger voltage pulse does not double the transmitted charge and hence the average current is sensitive to the exchange symmetry. Note that voltage pulses applied *across* the device are not expected to generate single particle excitations.

The discussion of full counting statistics in the presence of an energy-dependent scattering is difficult to conduct in a second-quantized formalism. On the other hand, the recent insight regarding the correspondence between fidelity and full counting statistics[13] has enabled a first-quantized approach based on a wave-packet formalism. Here, we make use of this new approach in our investigation of exchange effects on the full counting statistics of charge transport and find interesting new results: while it is expected that the noise as well as higher-order correlators will be sensitive to the exchange symmetry, we find the astounding result that exchange can hugely enhance (or suppress) the *average charge* already. The effect is a consequence of the symmetry-induced reshuffling of weight in the momentum distribution of the two-particle wave function, see Fig. 1, combined with the energy dependence of the scattering matrix: the antisymmetry under exchange moves weight away from the (Pauli-blocked) diagonal, which typically leads to an enhancement in the scattering channel transmitting one particle, at the expense of the channel where both particles are reflected. In a multi-channel/multi-lead setup the effect appears already for energy-independent transmission amplitudes.

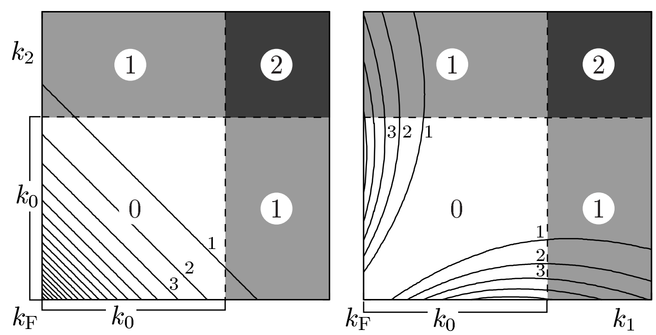

FIG. 1: Contour plots for the two-particle wave functions $|\Psi_{\mathrm{in},\pm}(k_1,k_2)|^2$ with vanishing separation δx between the particles. The three processes with 0, 1, and 2 particles transmitted draw their weight P_n, $n = 0, 1, 2$, from the (shaded) regions labelled with 0, 1, and 2 (shown is the case for the quantum point contact, with a Lorentzian wave function Eq. (7).)

II. SINGLE LEAD

A. First Quantized Picture

We start with the discussion of a quantum wire with one conducting channel, cf. Fig. 2(a). The task is to find the full counting statistics of the charge transport through a scatterer with an energy-dependent transmission amplitude located at $x = x_s$. Using the wave-packet formalism of Ref. 13, we calculate the generating function for the full counting statistics for two incoming particles, the simplest case exhibiting the effect of exchange symmetry. The incoming particles are described by two wave packets of the form

$$\psi_{\text{in},m}(x;t) = \int_0^\infty \frac{dk}{2\pi} f_m(k) e^{ik(x - v_\text{F} t)} \tag{1}$$

with the Fourier components $f_1(k)$ and $f_2(k)$. The normalization of the wave packets in momentum space reads $\int (dk/2\pi)|f_m(k)|^2 = 1$. At low temperatures the interesting physics takes place near the Fermi points and we can use a linearized spectrum $\epsilon = v_\text{F}|k|$ with the Fermi velocity v_F, the momentum $\hbar k$ and the energy $\hbar\epsilon$. Traversing the scatterer, the wave packet, Eq. (1), acquires a reflected and transmitted term

$$\psi^\sigma_{\text{out},m}(x,t) = \int_0^\infty \frac{dk}{2\pi} f_m(k) e^{-ikv_\text{F} t}\Big[r_k e^{-ikx}\Theta(x_s - x) + e^{i\sigma\lambda/2}\tau_k e^{ikx}\Theta(x - x_s)\Big], \tag{2}$$

where τ_k (r_k) is the transmission (reflection) amplitude and we have introduced a counting field $\exp(i\sigma\lambda/2)$ in the transmitted part; the sign $\sigma = \pm 1$ differentiates between the forward- and back-propagating wavefunctions required in the wave-packet formalism of full counting statistics, cf. Ref. 13. The properly (anti)symmetrized two-particle wave functions assume the form $\Psi_{\text{x},\pm}(x_1, x_2; t) \propto \psi_{\text{x},1}(x_1;t)\psi_{\text{x},2}(x_2;t) \pm (x_1 \leftrightarrow x_2)$ with x = in, out (see, Fig. 2(c); throughout the paper, the subscript '±' refers to the exchange symmetry, e.g., for two electrons this corresponds to the triplet or singlet states). The transport statistics is described by the generating function $\chi_\pm(\lambda) = \int dx_1 dx_2\, \Psi^{-1\,*}_{\text{out},\pm}\Psi^{+1}_{\text{out},\pm}$ and we obtain the result

$$\chi_\pm(\lambda) = \frac{\big[1 + (e^{i\lambda} - 1)\langle 1|T|1\rangle\big]\big[1 + (e^{i\lambda} - 1)\langle 2|T|2\rangle\big]}{1 \pm |S|^2} \pm \frac{\big[S + (e^{i\lambda} - 1)\langle 1|T|2\rangle\big]\big[S^* + (e^{i\lambda} - 1)\langle 2|T|1\rangle\big]}{1 \pm |S|^2} \tag{3}$$

with the matrix elements $\langle n|T|m\rangle = \int (dk/2\pi) f_n^*(k) T_k f_m(k)$ involving the transmission probability $T_k = |\tau_k|^2$ and the overlap integral $S = \int (dk/2\pi) f_1^*(k) f_2(k)$.[13] The Fourier transform $P_n = \int (d\lambda/2\pi)\chi(\lambda)e^{-i\lambda n}$ yields the probability P_n for transmitting n particles.

For *energy-independent* transmission amplitudes $\tau_k \equiv \tau$, the exchange term in Eq. (3) cancels with the denominator and the counting statistics does not dependent on the exchange symmetry of the particles. This property is particular to the one-dimensional wire; in a multi-lead setup, this cancellation does not occur and interference terms can be observed even for energy-independent scattering amplitudes, see below.

On the contrary, for an *energy-dependent* transmission, the exchange term has a dramatic effect on the

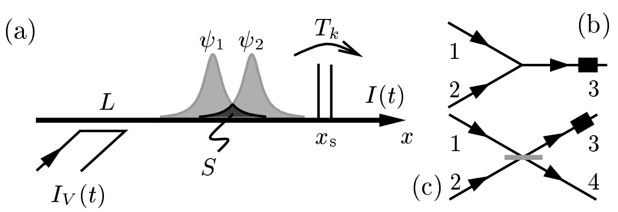

FIG. 2: (a) Single channel quantum wire driven inductively (L) with a nearby current $I_V(t)$. Two Lorentzian voltage pulses $V(t) = L\dot{I}_V/c$ drive the wave packets ψ_1 and ψ_2 with overlap S. The scatterer at $x = x_s$ gives rise to a momentum-dependent transmission probability T_k. (b) Three lead fork geometry with incoming wave packets in leads 1 and 2 and the counter (black box) in lead 3. (c) Reflectionless four-lead beam splitter; the incoming wave packets in leads 1 and 2 undergo a reflectionless transmission into leads 3 and 4.

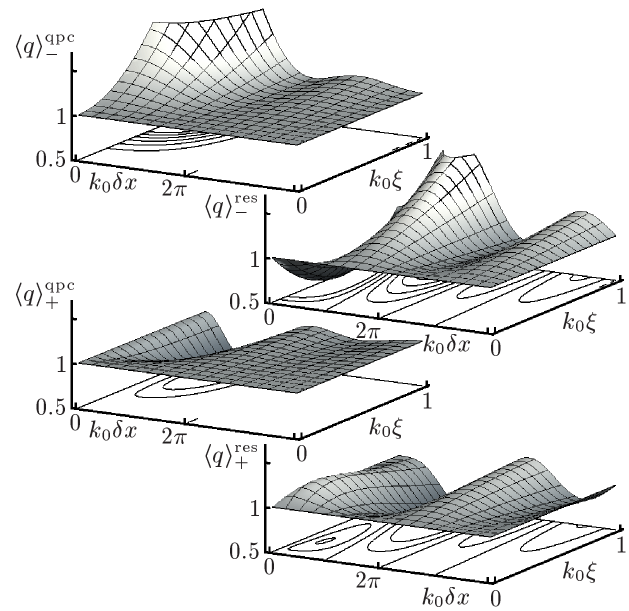

FIG. 3: Normalized average charge $q = Q/2e\langle T\rangle$ transmitted through a transmission step (left) and a transmission resonance (right). The step/resonance is placed at a distance k_0 away from the Fermi momentum k_F, ξ is the width of the wave packets in real space, and δx denotes the separation between the wave packets. The top (bottom) figures show the average transmitted charge for particles with negative (positive) exchange symmetry. Note the huge enhancement in the average transmitted charge in the limit $\delta x \to 0$ and $k_0\xi \geq 1$ for the case of negative exchange.

statistical properties of the transferred charge. To keep the discussion simple, we consider the case of two incident wave packets with the same momentum distribution separated in position. Given the initial separation δx, the Fourier components of the wave packets satisfy $f_2(k) = f_1(k)e^{-ik\delta x}$ and thus, $\langle 1|T|1\rangle = \langle 2|T|2\rangle \equiv \langle T\rangle = \int (dk/2\pi) T_k |f_1(k)|^2$. The overlap integral $S = \int (dk/2\pi)|f_1(k)|^2 \exp(ik\delta x)$ is the Fourier transform of the distribution in momentum space. The transmission probabilities $P_{n,\pm}$ are given by ($\sum_n P_{n,\pm} = 1$)

$$\begin{aligned} P_{0,\pm} &= \frac{(1-\langle T\rangle)^2 \pm |S - \langle 1|T|2\rangle|^2}{1 \pm |S|^2}, \\ P_{1,\pm} &= 2\frac{\langle T\rangle(1-\langle T\rangle) \pm \big[\mathrm{Re}(\langle 1|T|2\rangle S^*) - |\langle 1|T|2\rangle|^2\big]}{1 \pm |S|^2}, \\ P_{2,\pm} &= \frac{\langle T\rangle^2 \pm |\langle 1|T|2\rangle|^2}{1 \pm |S|^2}; \end{aligned} \tag{4}$$

they depend on the exchange symmetry provided that $\langle 1|T|2\rangle \neq S\langle T\rangle$. These transmission probabilities are easily converted into cumulants of transmitted charge $Q = \int dt I(t)$.[14] For two particles incident on the scatterer, the first two cumulants read (e denotes the charge of the particles)

$$\begin{aligned} \langle Q/e\rangle_\pm &= P_{1,\pm} + 2P_{2,\pm} \\ \langle\langle (Q/e)^2\rangle\rangle_\pm &= P_{1,\pm}(1-P_{1,\pm}) + 4P_{2,\pm}P_{0,\pm} \end{aligned} \tag{5}$$

and we find both depending on the exchange symmetry. While interference terms in the noise $\langle\langle Q^2\rangle\rangle$ due to the exchange symmetry of electrons are expected and have been found before,[10,11,12] here, we find that already the transmitted charge $\langle Q\rangle$ is sensitive to the exchange symmetry.

For a quantitative analysis of the effect, we need to specify the shape f_1 of the wave packets as well as the energy dependence T_k of the transmission probability. Rather than postulating trivial Gaussian wave packets, here, we consider a more realistic situation where the wave packets are deliberately created by voltage pulses. The creation of such single-particle excitations requires voltage pulses of unit flux[6] $\Phi_0 = hc/e$ and with Lorentzian shape,[9] $V_{t_1}(t) = -(2v_{\mathrm{F}}\xi\Phi_0/c)/[v_{\mathrm{F}}^2(t-t_1)^2 + \xi^2]$, where we conveniently express the pulse duration ξ/v_{F} through the length parameter ξ. Such a voltage pulse generates a wave packet with amplitude ($x_1 = v_{\mathrm{F}}t_1$)

$$f_1(k) = \sqrt{4\pi\xi}\, e^{-\xi(k-k_{\mathrm{F}}) - ikx_1}\Theta(k - k_{\mathrm{F}}), \tag{6}$$

and a Lorentzian shape in real space,

$$|\psi_1|^2 = \frac{\xi/\pi}{(x - x_1 - v_{\mathrm{F}}t)^2 + \xi^2}, \tag{7}$$

cf. Appendix A.

For two wave packets separated by δx we obtain an overlap integral $S = e^{-ik_{\mathrm{F}}\delta x}/(1 + i\delta x/2\xi)$.

Next, we concentrate on the scatterer where we consider two generic types: (i) for a sharp *transmission resonance*, e.g., due to a double barrier, the energy dependent transmission probability assumes the form $T_k^{\mathrm{res}} = \alpha/[1 + \beta^2(k - k_{\mathrm{F}} - k_0)^2]$ where $\alpha \leq 1$ is the amplitude of the resonance and $k_0 > 0$ is its position relative to the Fermi wave vector k_{F}. The width β^{-1} of the resonance has to be much smaller than the width ξ^{-1} of the wave packet in k-space, $\beta^{-1} \ll \xi^{-1}$. The transmission probability $\langle T^{\mathrm{res}}\rangle$ for a single wave packet with amplitude $f_1(k)$ assumes the form $\langle T^{\mathrm{res}}\rangle \approx (2\pi\alpha\xi/\beta)e^{-2\xi k_0}$ for $\beta^{-1} \ll k_0$, i.e., a resonance away from the Fermi level. While a small parameter $k_0\xi$ produces a large overall signal, it does suppress the effect of exchange symmetry since the transmission is already saturated, hence, below we will be interested in intermediate and large values of $k_0\xi$. (ii) For a (sharp, $\beta^{-1} \ll \xi^{-1}$) *transmission step*, e.g., due to a quantum point contact, we use the Kemble function[15] $T_k^{\mathrm{qpc}} = \alpha/[1+e^{-\beta(k-k_{\mathrm{F}}-k_0)}]$ producing the averaged transmission probability $\langle T^{\mathrm{qpc}}\rangle \approx \alpha e^{-2\xi k_0}$; note that the small prefactor ξ/β is absent in this case.

For a sharp resonance, the exchange term assumes the simple form $\langle 1|T^{\mathrm{res}}|2\rangle \approx e^{-i(k_{\mathrm{F}}+k_0)\delta x}\langle T^{\mathrm{res}}\rangle$ and its product with the overlap integral S^* results in an expression $\propto \exp(-ik_0\delta x)$ independent of k_{F}. The average transmitted charge, cf. Fig. 3, then exhibits oscillations in the separation δx,

$$\begin{aligned} \langle Q/e\rangle_\pm^{\mathrm{res}} &= 2\langle T^{\mathrm{res}}\rangle \\ &\times \frac{1 + (\delta x/2\xi)^2 \pm [\cos(k_0\delta x) + (\delta x/2\xi)\sin(k_0\delta x)]}{1 + (\delta x/2\xi)^2 \pm 1}. \end{aligned} \tag{8}$$

For wave packets with a large separation $\delta x \gg \xi$, the exchange term decays as $(\delta x)^{-2}$ and the transmitted charge is given by $\langle Q/e\rangle^{\mathrm{res}} = 2\langle T^{\mathrm{res}}\rangle$, independent on the sign of the exchange term. On the other hand, strongly overlapping wave packets with $\delta x \to 0$ reproduce the result for independent particles $\langle Q/e\rangle_+^{\mathrm{res}} = 2\langle T^{\mathrm{res}}\rangle$ for the symmetric exchange. For the anti-symmetric case the transmitted charge

$$\langle Q/e\rangle_-^{\mathrm{res}} = 2\langle T^{\mathrm{res}}\rangle(1 - 2\xi k_0 + 2\xi^2 k_0^2) \tag{9}$$

is reduced for narrow wave packets, $\xi k_0 < 1$ and enhanced for wave packets with $\xi k_0 > 1$; the reduction can be up to 50% for $\xi k_0 = 1/2$, while the increase is unlimited for $\xi k_0 > 1$.[16] Note that this increase is due to a large $P_{1,-}$ as $P_{2,-}$ vanishes.

The quantum point contact yields almost identical results: The off-diagonal matrix element assumes the form $\langle 1|T^{\mathrm{qpc}}|2\rangle \approx e^{-ik_0\delta x}S\langle T^{\mathrm{qpc}}\rangle$ and the *interference term* in P_2 vanishes. The transmitted charge

$$\langle Q/e\rangle_\pm^{\mathrm{qpc}} = 2\langle T^{\mathrm{qpc}}\rangle \frac{1 + (\delta x/2\xi)^2 \pm \cos(k_0\delta x)}{1 + (\delta x/2\xi)^2 \pm 1} \tag{10}$$

again exhibits oscillations with $k_0\delta x$, cf. Fig. 3. The various limits discussed above are reproduced as well, except for the case of anti-symmetric exchange and strongly

overlapping wave packets, cf. (9), where the average transmitted charge now is given by

$$\langle Q/e\rangle_-^{\rm qpc} = 2\langle T^{\rm qpc}\rangle(1+2\xi^2k_0^2). \qquad (11)$$

Eqs. (9) and (11) are the most striking results of our study: for a large parameter ξk_0 the mean transmitted charge can be hugely increased as compared to the value expected for two independent wave packets. Note that the above results are valid in a regime where all relevant lengths remain below the phase breaking length L_φ. Finally, we provide the expressions for the generating function of the full counting statistics,

$$\begin{aligned}\chi_\pm^{\rm res} =& 1+\langle Q/e\rangle_\pm^{\rm res}(e^{i\lambda}-1) &(12)\\ &+\langle T^{\rm res}\rangle^2\frac{(1\pm 1)[(\delta x/2\xi)^2+1]}{(\delta x/2\xi)^2+1\pm 1}(e^{i\lambda}-1)^2,\\ \chi_\pm^{\rm qpc} =& 1+\langle Q/e\rangle_\pm^{\rm qpc}(e^{i\lambda}-1)+\langle T^{\rm qpc}\rangle^2(e^{i\lambda}-1)^2. &(13)\end{aligned}$$

The above enhancements in $\langle Q\rangle_-$ are entirely due to P_1; there are two crucial elements in the game, Pauli exclusion and dispersive scattering. Dispersive scattering generates broader wave functions and combined with Pauli exclusion we have a reduction in P_0 and P_2, and hence an increase in P_1. However, different settings may be thought of where P_2 is enhanced. E.g., a large $P_{2,-}$ (see (3)) is obtained for wave packets with shifted amplitudes $f_2(k) = f_1(k+\delta k)$ in k-space and a large overlap integral S combined with a transmission amplitude suppressing k-values in the overlap region. A large enhancement in the tunneling probability of two identical particles due to an enhanced P_2 was also found by Suslov and Lebedev.[17]

B. Second Quantized Picture

In a second quantized picture, electronic excitations in the quantum wire are produced by a voltage pulse $V(t)$ applied at $x=0$.[18] If the time dependence of the voltage is slow compared to the transition time of an electron through the loop, the potential can be considered as quasistatic. It can then be incorporated in the phase factor $\exp[i\phi(t-x/v_{\rm F})\Theta(x)]$, where $v_{\rm F}$ is the Fermi velocity and the phase,

$$\phi(t) = \frac{e\Phi(t)}{\hbar c} = -\frac{e}{\hbar}\int_{-\infty}^t dt' V(t'), \qquad (14)$$

is proportional to the flux $\Phi(t)$. The solution $\psi_{{\rm L},k}(x;t) = \exp[ik(x-v_{\rm F}t)+i\phi(t-x/v_{\rm F})\Theta(x)]$ of the time dependent Schrödinger equation[19] is a plane wave with well-defined energy $\hbar v_{\rm F}k$, left of the position of the voltage pulse, $x<x_{\rm s}$. The wave function $\psi_{{\rm L},k}(x;t)$ for $x>0$ can be decomposed into energy eigenmodes $\exp[ik(x-v_{\rm F}t)]$ of the free Hamiltonian

$$\psi_{{\rm L},k}(x_{\rm s}>x>0;t) = \int\frac{dk'}{2\pi}U(k'-k)e^{ik'(x-v_{\rm F}t)} \qquad (15)$$

where the transformation kernel

$$U(q) = v_{\rm F}\int dt e^{i\phi(t)+iqv_{\rm F}t} \qquad (16)$$

is the Fourier transform of the phase factor $\exp[i\phi(t)]$. In the present work, we are interested in applying integer flux pulses such that $\phi(t\to\infty)\in 2\pi\mathbb{N}$ as noninteger flux pulses do not produce clean single-particle excitations and lead to logarithmic divergences in the noise of the transmitted charge.[6,20] In the case of integer flux pulses, the behavior of $\exp[i\phi(t)]\to 1$ for large times, $t\to\pm\infty$, leads to a Dirac delta function $2\pi\delta(q)$. The remaining part $U^{\rm reg}(q) = U(q)-2\pi\delta(q)$ is finite for a localized voltage pulse;

$$U^{\rm reg}(q) = v_{\rm F}\int dt\big[e^{i\phi(t)}-1\big]e^{iqv_{\rm F}t}. \qquad (17)$$

The transformation $U(k'-k)$ describes the scattering amplitude for the transition from a momentum state k (for $x<0$) to the state k' (for $x>0$) due to the application of the voltage pulse. The statement that the wave function is in the momentum state k' holds in the asymptotic region, i.e., for observation points with $x_{\rm obs}\gg\xi$. The scattering matrix approach at the scatterer assign the momentum component k' a transmission amplitude $\tau_{k'}$ for traversing the scatterer. Therefore, momentum eigenstates k incoming from the left assume the form

$$\psi_{{\rm L},k}(x>x_{\rm s},t) = \int\frac{dk'}{2\pi}U(k'-k)e^{ik'(x-v_{\rm F}t)}\tau_{k'}, \qquad (18)$$

right of the scattering region. The states originating from the right do not enter the region where the voltage pulse is applied and consists of the incoming and the reflected part

$$\psi_{{\rm R},k}(x>x_{\rm s},t) = (e^{-ikx}+r_ke^{ikx})e^{-ikv_{\rm F}t} \qquad (19)$$

without any shift in energy. The time dependent field operator is given by

$$\Psi(x>x_{\rm s},t) = \int\frac{dk}{2\pi}\big[\psi_{{\rm L},k}(x;t)a_k+\psi_{{\rm R},k}(x;t)b_k\big] \qquad (20)$$

in the region $x>x_{\rm s}$ behind the scatterer; here, a_k and b_k denote fermionic annihilation operators for states coming from the left, right reservoir, respectively. Averaging the current operator, defined as $I(x>x_{\rm s};t) = e\hbar\big(\Psi^\dagger(x;t)\partial_x\Psi(x;t)-[\partial_x\Psi^\dagger(x;t)]\Psi(x;t)\big)/2im$, over the Fermi reservoirs assuming the same Fermi distribution $n(k)$ at the far right and left of the interaction region and integrating the ensemble averaged current over time, the average transmitted charge

$$\langle Q/e\rangle = \int\frac{dk}{2\pi}n(k)\int\frac{dk'}{2\pi}K(k'-k)T_{k'} \qquad (21)$$

is obtained. It depends through the kernel $K(q) = |U^{\rm reg}(q)|^2+4\pi\delta(q){\rm Re}\big[U^{\rm reg}(0)\big]$ on different transmission

probabilities T_{k+q} than incoming wave vector k; note that for $U^{\rm reg}(q)$ whose real part is not continuous near $q=0$, $\mathrm{Re}[U^{\rm reg}(0)]$ has to be replaced by the symmetric limit $\mathrm{Re}[U^{\rm reg}(0+i\varepsilon)+U^{\rm reg}(0-i\varepsilon)]/2$ ($\varepsilon\to 0$). Equivalently, the kernel $K(q)$ can be defined as

$$K(q)=v_{\rm F}^2\int dtdt'\big[e^{i\phi(t)-i\phi(t')}-1\big]e^{iqv_{\rm F}(t-t')}. \tag{22}$$

The first quantized result, $\langle Q/e\rangle=\int(dk/2\pi)|f(k)|^2T_k$, is the same as the second quantized, Eq. (21), provided that $\int(dk'/2\pi)n(k')K(k-k')=|f(k)|^2$, where $f(k)$ denotes the wave function in the first quantized picture.

In a first step, we discuss a unit flux voltage pulse of Lorentzian shape $V_{t_1}(t)=-(2v_{\rm F}\xi\hbar/e)/[v_{\rm F}^2(t-t_1)^2+\xi^2]$, $v_{\rm F}\xi\gg 1$ applied at time t_1. The regular part of the transformation kernel is given by

$$U^{\rm reg}_{x_1}(q)=-2\pi(2\xi)e^{-\xi q-iqx_1}\Theta(q), \tag{23}$$

where $x_1=v_{\rm F}t_1$. The Θ-function reflects the fact that the Lorentzian pulse only increases the energy of the individual Fourier components. As we will discuss in Appendix A, the pulse produces a clean single particle excitation on top of the Fermi sea. Calculating

$$K_{x_1}(q)=(2\pi)^2 2\xi\Big[2\xi e^{-2\xi q}\Theta(q)-\delta(q)\Big] \tag{24}$$

and inserting the result in Eq. (21) yields the average charge transmitted, (for zero temperature with $n(k)=\Theta(k_{\rm F}-k)$, i.e., temperatures $\vartheta\ll\hbar\xi/v_{\rm F}$)

$$\begin{aligned}\langle Q/e\rangle_{x_1}=&\int_0^{k_{\rm F}}dk\int dk'2\xi\Big[2\xi e^{-2\xi(k'-k)}\Theta(k'-k)\\&-\delta(k'-k)\Big]T_{k'}=\int\frac{dk}{2\pi}|f_1(k)|^2T_k.\end{aligned} \tag{25}$$

In the last step, we performed the integration over k and renamed the remaining variable of integration. Furthermore, we dropped terms with $k\approx 0$. These terms correspond to the fact that the left states are shifted up in energy and therefore, currents from filled states on the right for small k values are not canceled by corresponding currents on the left. This is an unphysical artifact of the linear spectrum approximation which is only valid close to the Fermi point, $k\approx k_{\rm F}$. Formally, one can cure the problem by setting the lower integration bound for the k integration to $-\infty$ whenever one integrates over the Fermi sea. The empty states are then shifted to $k\to-\infty$ and do not appear in the final result. Alternatively, the transmission probability T_k can be set to 0 for small values of k. Eq. (25) is exactly the first quantized result for a wave packet of Lorentzian form, Eq. (6). More interesting than the single voltage pulse are two pulses separated by $\delta x=v_{\rm F}\delta t=x_2-x_1$ as they produce two wave packets and we expect interference terms to be found as in the first quantized formalism. For two pulses, the phase factor is a product of the phase factors of the individual pulses. Therefore, the Fourier transformed function $U_{x_1,x_2}(q)$ is the convolution of the transformation functions for single pulses $U_{x_1,x_2}(q)=U_{x_1}{*}U_{x_2}(q)$,

$$\begin{aligned}U^{\rm reg}_{x_1,x_2}(q)-U^{\rm reg}_{x_1}(q)-U^{\rm reg}_{x_2}(q)&=U^{\rm reg}_{x_1}{*}U^{\rm reg}_{x_2}(q)\\&=\frac{2(2\pi)(2\xi)^2}{\delta x}e^{-\xi q-iq(x_1+x_2)/2}\sin(q\delta x/2)\Theta(q).\end{aligned} \tag{26}$$

In the expression for the transmitted charge, we need to know the kernel K given by

$$\begin{aligned}K_{x_1,x_2}(q)=&K_{x_1}(q)+K_{x_2}(q)+2(2\pi)^2(2\xi)^2e^{-2\xi q}\Theta(q)\\&\times\left(2\frac{\big[\delta x\cos(q\delta x/2)-2\xi\sin(q\delta x/2)\big]^2}{(\delta x)^2}-1\right).\end{aligned} \tag{27}$$

The first two terms correspond to the direct contribution, the last term describes the interference. At zero temperature the average transmitted charge

$$\begin{aligned}&\langle Q/e\rangle_{x_1,x_2}=\langle Q/e\rangle_{x_1}+\langle Q/e\rangle_{x_2}\\&+2\int_0^\infty dq\,2\xi e^{-2\xi q}T_{k_{\rm F}+q}\frac{1-\cos(q\delta x)-\frac{\delta x}{2\xi}\sin(q\delta x)}{(\delta x/2\xi)^2}\\&=P_{1,-}+2P_{2,-},\end{aligned} \tag{28}$$

for two Lorentzian voltage pulses separated by δx is obtained. When comparing Eq. (28) to the first quantized result (4), it can be seen that the result in second quantization is the same as expected from the analysis in the first quantized picture for an antisymmetric wave function. For a single voltage pulse with double flux, i.e. $\delta x=0$, the increase in the transmitted charge as seen in the first quantized formalism is confirmed. Moreover, applying a voltage pulse carrying even more flux quantum, leads to a further increase in the transmitted charge with respect to the independent particle picture, cf. Appendix B.

So far, we considered only spinless particles. The electronic spin 1/2 is included easily in the second quantized formalism. The summation over the spin degree of freedom provides a factor of two for each cumulant; i.e., each voltage pulse excites two particles in the same spatial state. In the first quantized formalism, a single voltage pulse which excites two particles in the state $\Psi_1(x;t)$ can be described as a singlet

$$\Psi(x_1,x_2,t)=\Psi_1(x_1,t)\Psi_1(x_2,t)\frac{|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle}{\sqrt{2}}. \tag{29}$$

Each of the two particles carries the same current and they are independent of each other which leads to a factor two as in the second quantized formalism. Accordingly, two unit flux voltage pulses excite a four particle state involving the wave functions $\Psi_{1,2}(x;t)$. The second quantized formalism leads a factor of two due to the spin degeneracy and an averaging over the energies of the antisymmetrized wave function $\Psi_-(x_1,x_2;t)=\big[\Psi_1(x_1;t)\Psi_2(x_2,t)-(x_1\leftrightarrow x_2)]\big]/\sqrt{2(1-|S|^2)}$. In first

quantization, the four-particle wave function with spin 0 is given by

$$\Psi(x_1,x_2,x_3,x_4,t) = \frac{1}{\sqrt{6}}\Big[\Psi_-(x_1,x_2,t)\Psi_-(x_3,x_4,t)\big(|\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle\big) + \Psi_-(x_1,x_4,t)\Psi_-(x_2,x_3,t)\big(|\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\downarrow\rangle\big) - \Psi_-(x_1,x_3,t)\Psi_-(x_2,x_4,t)\big(|\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle\big)\Big]. \tag{30}$$

As the spin is unimportant, we may integrate over the spin degrees of freedom and obtain a density matrix consisting of an equal mixture of the wave function $\Psi_-(x_1,x_2;t)\Psi_-(x_3,x_4;t)$ and wave functions obtained by coordinate relabeling. Relabeling does not change the physics of independent particles. We can choose any one of the wave functions in the first quantized picture. Both the antisymmetry (Ψ_-) and the trivial factor of two (product of two wave functions) are now also visible in the first quantized language. The generating function $\chi_\pm^{(4)} = {\chi_\pm}^2$ of the full counting statistics for the four spin degenerate particles is the square of the spinless generating functions $\chi_\pm$ of Eq. (12).

The procedure outline can also be carried out for different forms of the voltage pulse. Note though that for pulses other than a Lorentzian voltage pulse there is no simple first quantized correspondence to the second quantized formalism. A unit flux voltage pulse is given by

$$V(t) = -\frac{2\hbar\dot{f}(t)/e}{1+f(t)^2} \tag{31}$$

with the requirement that $f(t\to\pm\infty)\to\pm\infty$; the Lorentzian voltage pulse, considered so far, is a special case obtained with $f(t) = v_{\rm F}t/\xi$. The phase factor assumes the form

$$e^{i\phi(t)} = \frac{f(t)-i}{f(t)+i}. \tag{32}$$

As an example we consider the voltage pulse given by $f(t) = (v_{\rm F}t/\xi)^3$ which has a simple transformation function $U(q) = 2\pi\delta(q) + U^{\rm reg}(q)$ with

$$U^{\rm reg}(q) = -2\pi\frac{2\xi}{3}\big[e^{\xi q}\Theta(-q) + e^{-\xi q/2}[\cos(\sqrt{3}\xi q/2) + \sqrt{3}\sin(\sqrt{3}\xi q/2)]\Theta(q)\big] \tag{33}$$

The part with positive momentum transfer, $q>0$, corresponds to an excited electron as we have seen for the simple Lorentzian pulse. The negative momentum indicates holes accompanying the electron. The kernel K assumes the form

$$K(q) = (2\pi)^2\frac{(2\xi)^2}{9}\big[e^{2\xi q}\Theta(-q) + e^{-\xi q}[2-\cos(\sqrt{3}\xi q) + \sqrt{3}\sin(\sqrt{3}\xi q)]\Theta(q)\big] - 2(2\pi)^2(2\xi)\delta(q)/3. \tag{34}$$

Plugging it in Eq. (21), yields the average transmitted charge

$$\langle Q/e\rangle = \frac{2\xi}{9}\int dk\, T_k\Big[-e^{-2\xi(k_{\rm F}-k)}\Theta(k_{\rm F}-k) + e^{-\xi(k-k_{\rm F})}\Big(4+\cos\big[\sqrt{3}\xi(k-k_{\rm F})\big] + \sqrt{3}\sin\big[\sqrt{3}\xi(k-k_{\rm F})\big]\Big)\Theta(k-k_{\rm F})\Big] \tag{35}$$

at zero temperature. The part with the negative momentum transfer carries a negative charge, hole, and the charge of the other part is positive and so the expected nature of excitations is confirmed. Note that in the case of a fully transparent wire ($T_k\equiv 1$), the average transmitted charge is exactly one electron e.

III. MULTI-LEAD

In a multi-lead setup, exchange effects can be found for an energy-independent scattering already. Here, we discuss the three-lead fork geometry and the four-lead reflectionless beam splitter for two incoming particles in different leads 1 and 2 and hence vanishing initial overlap. The wave packets are assumed to have a Lorentzian shape (6) and to impinge on the scatterer with a delay $\delta t = \delta x/v_{\rm F}$. In the three-lead geometry, cf. Fig. 2(b), the generating function reads

$$\chi_\pm^{\succ} = 1 + T_3(e^{i\lambda}-1) + (1\pm|S|^2)T_{13}T_{23}(e^{i\lambda}-1)^2, \tag{36}$$

where T_{13} and T_{23} denote the transmission probabilities for particles incident from leads 1 and 2 and propagating into lead 3 (containing the counter) and $T_3 = T_{13}+T_{23}$. The probabilities

$$\begin{aligned} P_{0,\pm}^{\succ} &= (1-T_{13})(1-T_{23}) \pm |S|^2T_{13}T_{23} \\ P_{1,\pm}^{\succ} &= T_{13}(1-T_{23}) + (1-T_{13})T_{23} \mp 2|S|^2T_{13}T_{23} \\ P_{2,\pm}^{\succ} &= (1\pm|S|^2)T_{13}T_{23} \end{aligned} \tag{37}$$

depend on the exchange symmetry as long as $T_{13}T_{23}\neq 0$. For $\delta x = 0$ the overlap integral $S = e^{-ik_{\rm F}\delta x}/(1+i\delta x/2\xi)$ becomes unity and hence $P_{2,-}=0$. Finally, the charge cumulants assume the form

$$\begin{aligned} \langle Q/e\rangle_\pm^{\succ} &= T_3, \\ \langle\langle (Q/e)^2\rangle\rangle_\pm^{\succ} &= T_{13}(1-T_{13}) + T_{23}(1-T_{23}) \pm 2|S|^2T_{13}T_{23}; \end{aligned} \tag{38}$$

effects due to exchange symmetry are limited to the charge noise which is enhanced (reduced) for the (anti-)symmetric case. For the reflectionless four-lead beam splitter with the counter in lead 3, Fig. 2(c), we obtain the generating function, probabilities, and moments in

the form

$$\begin{aligned}\chi_{\pm}^{\asymp} &= 1 + [T_3 \pm |S|^2(T_3 - 1)](e^{i\lambda} - 1)\\ &\quad + (1 \pm |S|^2)T_{13}T_{23}(e^{i\lambda} - 1)^2\\ P_{0,\pm}^{\asymp} &= (1 \pm |S|^2)(1 - T_{13})(1 - T_{23})\\ P_{1,\pm}^{\asymp} &= (1 \pm |S|^2)[(1 - T_{13})T_{23} + T_{13}(1 - T_{23})]\\ &\quad \mp |S|^2\\ P_{2,\pm}^{\asymp} &= (1 \pm |S|^2)T_{13}T_{23}\\ \langle Q/e\rangle_{\pm}^{\asymp} &= (1 \pm |S|^2)T_3 \mp |S|^2\\ \langle\langle (Q/e)^2\rangle\rangle_{\pm}^{\asymp} &= (1 \pm |S|^2)\big[(1 - T_{13})T_{13} + (1 - T_{23})T_{23}\\ &\quad \mp |S|^2(T_3 - 1)^2\big]. \end{aligned} \tag{39}$$

Particles with anti-symmetric exchange and $\delta x = 0$ exhibit no partitioning and the noise vanishes,[12] as $P_{0,-}^{\asymp} = P_{2,-}^{\asymp} = 0$ and $P_{1,-}^{\asymp} = 1$, i.e., each of the incoming particles is transmitted in a different outgoing lead.[12] In the nonsymmetric case $T_3 \neq 1$ (note that $T_{13} + T_{14} = 1$ and $T_{14} = T_{23}$ in the symmetric situation), an exchange effect shows up already in the average transmitted charge $\langle Q/e\rangle_{\pm}^{\asymp} = (1 \pm |S|^2)T_3 \mp |S|^2$.

IV. EXPERIMENTAL VERIFICATION

In order to observe exchange effects in an experiment, we propose to test for the predicted non-linearity in the transport: comparing the average transmitted charge $\langle Q^{(1)}\rangle$ for a single flux pulse with the one ($\langle Q^{(2)}\rangle$) injected with doubled voltage, our analysis predicts the non-trivial result $\langle Q^{(2)}\rangle/2\langle Q^{(1)}\rangle = 1 + 2\xi^2 k_0^2$, cf., Eqs. (11); for a transmission resonance, $\langle Q^{(2)}\rangle/2\langle Q^{(1)}\rangle = 1 - 2\xi k_0 + 2\xi^2 k_0^2$, cf., Eqs. (9). Given a pulse length of duration $\sim 10^{-10}$ s, such an experiment requires a device with a phase coherence length beyond 10 μm and is preferably carried out on a quantum point contact where Coulomb effects are less prominent. In this case, we have to compare the spread in energy $\hbar v_{\rm F}/\xi$ of the wave packet with the charging energy e^2/C, where $C \sim \varepsilon L$ denotes the junction capacitance (L the length of the point contact). For a sharp transmission step, we have $\xi/L < 1$ small and the relation $\xi/L < \varepsilon(\hbar c/e^2)(v_{\rm F}/c)$ can be satisfied.

V. CONCLUSION

In summary, we have studied effects of the exchange symmetry on the transport statistics in mesoscopic systems. We find a strong dependence (and possibly huge enhancement) of the average charge transmission on the exchange symmetry, provided that the wave function sufficiently overlap. For antisymmetric exchange such overlap pushes weight of the two-particle wave function to higher energies and combined with the energy dependent scattering this leads to the observed enhancement. For multi-channel/multi-lead setups, exchange effects can already be observed for energy-independent scattering probabilites. There the origin of the effect is rooted in simple Pauli-blocking. Furthermore, we have proposed a experiment to test for the most striking effect of exchange; the non-linearity in transport.

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APPENDIX A: EQUIVALENCE BETWEEN THE FIRST AND SECOND QUANTIZED PICTURE

Finally, we motivate the form (6) for the k-space amplitude $f_1(k)$ of the pulse-generated wave packet. In the adiabatic limit, a voltage pulse $V(t)$ applied at $x = 0$ adds a phase $\phi(t) = -(e/\hbar)\int_{-\infty}^{t} dt' V(t')$, $e > 0$, to the wave function across the origin[19]. We derive the scattering matrix $U_{k',k}$ describing the transitions from k states at $x < 0$ to k' states at $x > 0$. This is conveniently done in real space, where a particle described by the time-retarded state $|\tilde{x}\rangle$ with $\langle x, t|\tilde{x}\rangle = \delta(x - v_{\rm F}t)$ for $x < 0$ evolves to $\exp[i\phi(-\tilde{x}/v_{\rm F})]|\tilde{x}\rangle$ at $x > 0$ under the action of the voltage pulse ($\tilde{x} = x - v_{\rm F}t$ is the retarded variable). An arbitrary incoming state at $x < 0$ then is transformed into the corresponding outgoing state at $x > 0$ via the unitary scattering operator $\hat{U} = \exp[i\int d\tilde{x}\phi(-\tilde{x}/v_{\rm F})\Psi^\dagger(\tilde{x})\Psi(\tilde{x})]$ with the time-retarded field operator $\Psi(\tilde{x})$. We introduce the fermionic annihilation operators a_k, generating basis states $\exp(ik\tilde{x})$, and transform to k-space via $\Psi(\tilde{x}) = \int(dk/2\pi)\exp(ik\tilde{x})a_k$ to find the matrix elements $U_{k',k} = U(k' - k) = v_{\rm F}\int dt \exp[i\phi(t) + i(k' - k)v_{\rm F}t]$. The scattering amplitude then relates to the amplitude $f_1(k)$ in (6) via $U_{x_1}(q) = -\sqrt{4\pi\xi}f_1(k_{\rm F} + q)\, e^{ik_{\rm F}v_{\rm F}t_1} + 2\pi\delta(q)$, cf. Eq. (23). The voltage pulse transforms the Fermi sea $|\Phi_{\rm F}\rangle$ to $|\bar{\Phi}_{\rm F}\rangle = \hat{U}_{x_1}|\Phi_{\rm F}\rangle$; the calculation of the density matrix at zero temperature

$$\begin{aligned}\langle\bar{\Phi}_{\rm F}|a_{k'}^\dagger a_k|\bar{\Phi}_{\rm F}\rangle &= \int\frac{dk''}{2\pi}U_{x_1}^*(k' - k'')U_{x_1}(k - k'')\Theta(k_{\rm F} - k'')\\ &= \langle\Phi_{\rm F}|a_{k'}^\dagger a_k|\Phi_{\rm F}\rangle + f_1^*(k')f_1(k), \end{aligned} \tag{A1}$$

involves the transformed annihilation operators

$$\hat{U}^\dagger a_k \hat{U} = a_k + \int\frac{dk'}{2\pi}U^{\rm reg}(k - k')a_{k'} \tag{A2}$$

with $U^{\rm reg}(q)$ defined in Eq. (17). It tells us that the voltage pulse leaves back a filled Fermi sea above which an excitation is created with an amplitude $f_1(k)$ given by Eq. (6). Furthermore, we find that the two-particle correlator $\langle\bar{\Phi}_{\rm F}|a_k^\dagger a_{k'}^\dagger a_{k'} a_k|\bar{\Phi}_{\rm F}\rangle$ vanishes for $k, k' > k_{\rm F}$ and hence pair-excitations are absent. To observe the effects of the exchange symmetry, the overlap integrals are important. To create two particles, two pulses need to be applied in sequence, $\hat{U} = \hat{U}_{x_2}\hat{U}_{x_1}$. A straightforward calculation shows that

$$\langle\Phi_{\rm F}|\hat{U}_{x_1}{}^\dagger\hat{U}_{x_2}{}^\dagger a^\dagger_{k'}a_k\hat{U}_{x_2}\hat{U}_{x_1}|\Phi_{\rm F}\rangle = \langle\Phi_{\rm F}|a^\dagger_{k'}a_k|\Phi_{\rm F}\rangle + \int\frac{dk''}{2\pi}\frac{[f_1^*(k')f_2^*(k'')-f_2^*(k')f_1^*(k'')][f_1(k)f_2(k'')-f_2(k)f_1(k'')]}{1-|S|^2},$$

i.e., the resulting state is the unperturbed Fermi system with an antisymmetrized two particle wave packet on top of it. Therefore, the first quantized picture can be used even for two voltage pulses resulting in a two particle wave function in the first quantized picture and Eq. (3) gives the full counting statistics of the two transmitted electrons.

APPENDIX B: MANY FLUX LORENTZIAN VOLTAGE PULSE

Applying a Lorentzian shaped n flux voltage pulse $V_n(t) = -(2nv_{\rm F}\xi\hbar/e)/[(v_{\rm F}t)^2+\xi^2]$, an even more drastic nonlinear effect is expected than found for the two particle case. The phase factor $\exp[i\phi(t)]$ is the n-th power of the single pulse phase factor. The Fourier transformation of the phase factor can be carried out using Cauchy's formula. We obtain

$$U_n^{\rm reg}(q) = 2\pi(2\xi)e^{-\xi q}\Theta(q)\sum_{l=1}^{n}\frac{n!(-1)^l(2\xi q)^{l-1}}{(l-1)!l!(n-l)!}. \qquad \text{(B1)}$$

The kernel is then given by

$$K_n(q) = -(2\pi)^2 n(2\xi)\delta(q) + (2\pi)^2(2\xi)^2 e^{-2\xi q}\Theta(q) \times \sum_{l,m=1}^{n}\frac{(n!)^2(-1)^{l+m}(2\xi q)^{l+m-2}}{(l-1)!l!(n-l)!(m-1)!m!(n-m)!} \qquad \text{(B2)}$$

To calculate the average transmitted charge, Eq. (B2) need to be integrated over the Fermi sea, c.f. Eq. (21). The averaging over the zero temperature Fermi sea leads to

$$|f(k)|_n^2 = \int\frac{dk'}{2\pi}n(k')K(k-k') = 2\pi(2\xi)\Theta(k-k_{\rm F}) \times \sum_{l,m=1}^{n}\frac{(n!)^2(-1)^{l+m}\Gamma(l+m-1,2\xi(k-k_{\rm F}))}{(l-1)!l!(n-l)!(m-1)!m!(n-m)!} \qquad \text{(B3)}$$

where $\Gamma(n,x_0) = \int_{x_0}^\infty dx\, x^{n-1}e^{-x}$ denotes the incomplete gamma function. The average charge transmitted is given by $\langle Q/e\rangle_n = \int(dk/2\pi)|f(k)|_n^2 T_k$. The result for the scattering resonance can then be calculated,

$$\begin{aligned}\langle Q/e\rangle_2 &= 2\langle T^{\rm res}\rangle(1-2\xi k_0+2\xi^2k_0^2)\\ \langle Q/e\rangle_3 &= 3\langle T^{\rm res}\rangle(1-4\xi k_0+8\xi^2k_0^2-16\xi^3k_0^3/3\\ &\quad +4\xi^4k_0^4/3)\\ \langle Q/e\rangle_4 &= 4\langle T^{\rm res}\rangle(1-6\xi k_0+18\xi^2k_0^2-68\xi^3k_0^3/2\\ &\quad +14\xi^4k_0^4-4\xi^5k_0^5+4\xi^6k_0^6/9), \qquad \text{(B4)}\end{aligned}$$

using the fact that the incomplete gamma function for integer n is expressible through a polynomial times an exponential. For the quantum point contact, the equivalent expressions read

$$\begin{aligned}\langle Q/e\rangle_2 &= 2\langle T^{\rm qpc}\rangle(1+2\xi^2k_0^2)\\ \langle Q/e\rangle_3 &= 3\langle T^{\rm qpc}\rangle(1+4\xi^2k_0^2-8\xi^3k_0^3/3+4\xi^4k_0^4/3)\\ \langle Q/e\rangle_4 &= 4\langle T^{\rm qpc}\rangle(1+6\xi^2k_0^2-8\xi^3k_0^3+22\xi^4k_0^4/3\\ &\quad -8\xi^5k_0^5/3+4\xi^6k_0^6/9). \qquad \text{(B5)}\end{aligned}$$

Increasing the voltage, $V_n \propto n$, which corresponds to sending more electrons through the scattering region, there is a huge nonlinear increase in the transmitted charge if $\xi k_0 \gg 1$.

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